Testing whether group-level fixed effects are sufficient in panel data models

David Vincent

dvincent@dveconometrics.co.uk

September 2025

2025 UK Stata Conference

Contents

Introduction

- The fixed effects (FE) estimator is one of the most widely used methods for estimating coefficients on time varying variables in linear panel data models.

- Its main appeal is that it controls for correlation between the regressors and unobserved unit specific effects that would otherwise lead to omitted variable bias.

- The FE estimator works by applying OLS to the within transformed data, where all of the variables are expressed as deviations from their unit specific means.

- This transformation removes between-unit (i.e. cross sectional), variation in the data, such that coefficients are estimated solely from within unit variation.

Limitations of Fixed Effects

- A well known limitation of FE is that when key variables of interest display little within-unit variation, the resulting estimates can be very imprecise.
- One approach is to turn to the random effects estimator, and decide whether this is valid by testing if the explanatory variables are uncorrelated with the unit-specific effects.
- The Hausman test is the conventional tool here, and as Wooldridge (2019) discusses, fully robust versions can be obtained from auxiliary regressions.
- Another possibility is to ask whether controlling for heterogeneity at a higher level of aggregation (e.g. group effects) might suffice to ensure regressors exogeneity.

Papke and Wooldridge (2023)

- If regressors are only correlated with group effects, both unit and group FE are consistent, but group FE may be more efficient. If regressors are correlated with unit effects, only unit FE is consistent and the estimates will differ.
- This is examined by Papke and Wooldridge (2023), who propose a variable-addition test: estimate the model by group FE, augmented with unit-means of the time-varying regressors, and test the null that all coefficients on the means are zero.
- This is the Mundlak (1978) regression, where the coefficients on the means are differences in the between-effects (controlling for group dummies) and the unit FE slopes, while the joint test is for equality of unit and group FE.

Papke and Wooldridge (2023)

- Although not discussed, the joint test is equivalent to a C-test of the orthogonality restrictions that justify group FE.
- Papke and Wooldridge (2023) also also propose a conventional Hausman test, which directly compares unit and group FE estimates for one regressors, and show how to obtain robust standard errors for the difference.
- While this allows one to focus on a specific coefficient of interest, the underlying estimators are unit and group FE.
- As such, the null can be rejected even when the variable of interest is uncorrelated with the unit-specific effect, provided it is correlated with other endogenous regressors.

Motivation for `xtfelevel`

- I present `xtfelevel`, which builds on Papke and Wooldridge (2023) by comparing unit and group fixed effects methods in a more general IV framework.
- Instead of the all-or-nothing approach, the test allows some time varying regressors to be treated as always endogenous in both the consistent and efficient estimators.
- If the null is not rejected, the IV estimator can be substantially more efficient than full unit FE, and is equivalent to group FE estimation of the Mundlak regression, where unit-means of exogenous variables are partialled out from unit means of endogenous regressors.
- This differs from Papke and Wooldridge (2023) hybrid estimator, which drops insignificant means from the regression and may introduce bias when regressors are correlated.

The Model

- To motivate the set-up, I use an instrumental variables (IV) framework that nests the standard unit and group fixed effects estimators as special cases.
- Let y_{it} denote the outcome for individual $i = 1, \ldots, N$ observed in period $t = 1, \ldots, T_i$, and let x_{1it} be a vector of time-varying regressors whose effects we are interested in.
- Individuals are assigned into G group, with group membership indicated by $g(i)$. The model to be estimated is:

$$y_{it} = x_{1it}^{'}\beta_1 + x_{2it}^{'}\beta_2 + h_{g(i)} + u_i + \epsilon_{it}, \qquad (1)$$

  where u_i is a unit-specific effect, $h_{g(i)}$ a group-specific effect, and ϵ_{it} an idiosyncratic error.

The Model

- The regressors x_{1it} are strictly exogenous with respect to ϵ_{it}, and are divided into two sets: x_{1Bit}, which are always assumed to be correlated with u_i, and x_{1Ait}, which may or may not be correlated with u_i.
- The variables in x_{2it} are typically time-invariant controls, but may also regressors that vary over time and are uncorrelated with u_i, such as macro-economic factors.
- All variables, including x_{2it} are allowed to be arbitrarily correlated with the group level heterogeneity $h_{g(i)}$ and the main focus on the and estimation of β_1.
- Note that even if x_{1A} is uncorrelated with u_i, the IV estimator that treats x_{1A} as exogenous may still differ from the one that treats it as endogenous when selection into the sample depends on both u_i and x_{1A}.

The Model

- It is helpful to consider the group-specific fixed effects as actual parameters to be estimated, which means that instruments are only needed for x_1.
- Let $h_{g(i)} = d_i^{\prime}\lambda$ where $d_i = \left[d1_{g(i)} ... dG_{g(i)}\right]^{\prime}$, is a vector of group dummy variables, such that $df_{g(i)} = 1$ if $f = g$. Then stacking (1) over all periods for individual i gives:

$$y_i = X_{1i}^{\prime}\beta_1 + X_{2i}^{\prime}\beta_2 + D_i\lambda + e_i u_i + \epsilon_i \tag{2}$$

where e_i is a $T_i \times 1$ vector of 1's, and $D_i = e_i d_i^{\prime}$ is a $T_i \times G$ matrix of observations on the dummy variables.

Hausman Test

- I consider two IV estimators that both treat the group-specific effects as fixed effects, but differ in how they control for correlation between x_{1it} and the unit-heterogeneity u_i.

- The first, which I shall refer to as the **consistent** estimator $\widehat{\beta}_1^c$ uses the unit-demeaned variables $x_{1it} - \bar{x}_{1i}$ as instruments and is consistent when x_{1it} is correlated with u_i or $h_{g(i)}$.

- The second $\widehat{\beta}_1^e$ again uses $x_{1Bit} - \bar{x}_{1Bi}$ as instruments for x_{1Bit}, but now uses x_{1Ait} as instruments for itself and will therefore be inconsistent when x_{1it} is correlated with u_i.

- When x_{1Ait} is uncorrelated with u_i, both estimators will have the same plim, but $\widehat{\beta}_1^e$ will tend to be more **efficient**, as the unit de-meaning employed by $\widehat{\beta}_1^c$ removes more of the variation in the data than is necessary.

Hausman Test

- This suggests testing the joint endogeneity of x_{1Ait} by testing for differences between the $\widehat{\beta}_1^c$ and $\widehat{\beta}_1^e$ estimators:

$$\begin{aligned} H_0 : \ \operatorname{plim} \widehat{\beta}_1^c - \operatorname{plim} \widehat{\beta}_1^e &= 0 \\ H_1 : \ \operatorname{plim} \widehat{\beta}_1^c - \operatorname{plim} \widehat{\beta}_1^e &\neq 0 \end{aligned} \tag{3}$$

- Under the null, the difference between the consistent and efficient IV estimators is also root-N consistent, with a mean 0 and a limit normal distribution, so that:

$$\sqrt{N}\left(\widehat{\beta}_1^c - \widehat{\beta}_1^e\right) \longrightarrow N\left[0, V\right]$$

- A Hausman test statistic is then:

$$H = \left(\widehat{\beta}_1^c - \widehat{\beta}_1^e\right)' \left(\widehat{V}/N\right)^{-1} \left(\widehat{\beta}_1^c - \widehat{\beta}_1^e\right) \xrightarrow{d} \chi^2_{K_{1A}}$$

Hausman Test

- Even though x_1 consists of K_1 variables, as only x_{1A} regressors are being tested, $\widehat{V}$ will be of reduced rank, equal to K_{1A}.
- The Hausman test can also be applied to to test for differences in a single coefficient for any of the parameters in x_1:

$$H_k = \left[\frac{\left(\widehat{\beta}^c_{1k} - \widehat{\beta}^e_{1k}\right)}{SE_k} \right]^2 \xrightarrow{d} \chi^2_1$$

  where $SE_k = \sqrt{\widehat{V}[k,k]/N}$ is the standard error of the k^{th} component of the difference. This could lead to a different conclusion from a test of all the parameters in β_1.
- The Hausman test is therefore best interpreted as a test of the consequences of different estimators on the same equation.

Valid Instruments

- The consistent estimator treats all variables as potentially correlated with $h_{g(i)}$, and x_1 as correlated with the u_i.
- Let $Q_i^u = I_{T_i} - M_i^u$ denote the within transformation matrix, which subtracts unit level means $M_i^u = T_i^{-1} e_i e_i^{'}$, then valid instruments for X_{1i} are $Q_i^u X_{1i}^{'}$, as then:

$$E\left[(Q_i^u X_{1i})^{'} (e_i u_i + \epsilon_i) \right] = E\left[X_{1i}^{'} Q_i^u \epsilon_i \right] = 0$$

- Stacking over all individuals, the instrument set is:

$$W_2 = [Q^u X, X_2, D] \tag{4}$$

where $Q^u = \text{diag}\{Q_1^u ... Q_N^u\}$ and the within transformed instruments also include the exogenous controls.

Consistent Estimation

- Basu (2023) shows that for IV estimators, exogenous variables can be partialled out of the instrument set and the model.
- For the set of G group dummies, D, this simply requires applying the within transformation at the group level.
- Letting $Q = \text{diag}\{Q_1 \ldots Q_G\}$ denote the equivalent transformation matrix by group, then the within transformed model subtracting group means is:

$$\tilde{y} = \tilde{X}_1' \beta_1 + \tilde{X}_2' \beta_2 + \tilde{v} \tag{5}$$

where $\tilde{y} = Qy$, $\tilde{X} = QX$ and the error $\tilde{v} = Qv$, where for individual i, this is defined above as $v_i = e_i u_i + \epsilon_i$.

Consistent Estimation

- As units are nested within groups, the group mean of the already unit-demeaned data is zero, hence $Q \times Q^u = Q^u$. Thus, after partialling out D from the instruments:

$$W_2 = \left[Q^u X, \tilde{X}_2 \right] \tag{6}$$

- Let $P_2 = W_2(W_2' W_2)^{-1} W_2'$ denote the projection matrix, and $M_{\tilde{X}_2}$ the usual residual maker, then by the FWL theorem, the IV estimator that treats all variables in x_1 as exogenous is:

$$\widehat{\beta}_1^c = (\tilde{X}_1' P_2 M_{\tilde{X}_2} P_2 \tilde{X}_1)^{-1} \tilde{X}_1 P_2 M_{\tilde{X}_2} \tilde{y} = A_c^{-1} \tilde{X}_1 P_2 M_{\tilde{X}_2} \tilde{y} \tag{7}$$

Consistent Estimation

- It is helpful to note that the predicted values $P_2\tilde{X}_1$ can be decomposed into within and between-unit components:

$$P_2\tilde{X}_1 \;=\; Q^u X_1 \;+\; P_{M^u\tilde{X}_2} M^u\tilde{X}_1. \tag{8}$$

- The first term is the within-unit variation in X_1, whereas the second term, are the predictions from separate between-effects regressions of x_{1kit} on x_{2it}, including G group dummies
- This is equivalent to running OLS on the unit-level means of these variables, with G group dummies included, where each mean is replicated T_i times in the dataset.

Consistent Estimation: Unit Fixed Effects

- This shows that the variation used to estimate β_1 includes not only the within-unit variation in X_1, but also the between-unit variation that is predicted by the exogenous controls X_2.
- In most applications, the controls X_2 are time-invariant, and when this is the case, the additional between variation makes no contribution, and is eliminated by $M_{\tilde{X}_2}$.
- Also, as $M_{\tilde{X}_2} Q^u = Q^u$ this IV estimator that treats all x_1 as endogenous, reduces to standard unit fixed effects:

$$\widehat{\beta}_1^c = \widehat{\beta}_1^{FE(i)} \left(X_1' Q^u X_1 \right)^{-1} X_1' Q^u y.$$

Efficient Estimation

- For the more efficient IV estimator, X_{1A} is now treated as exogenous, with only X_{1B} as endogenous, hence the instrument set becomes:

$$W_1 = \left[Q^u X, \tilde{X}_{1A}, \tilde{X}_2 \right] \qquad (9)$$

- Defining P_1 as the projection onto W_1 yields:

$$\widehat{\beta}_1^e = \left(\tilde{X}_1' P_1 M_{\tilde{X}_2} P_1 \tilde{X}_1 \right)^{-1} \tilde{X}_1' P_1 M_{\tilde{X}_2} \tilde{y} = A_e^{-1} \tilde{X}_1' P_1 M_{\tilde{X}_2} \tilde{y} \quad (10)$$

- When all variables are tested, $\tilde{X}_{1A} = \tilde{X}_1$, and the estimator reduces to group fixed effects:

$$\widehat{\beta}_1^e = \widehat{\beta}_1^{FE(g)} = \left(\tilde{X}_1' M_{\tilde{X}_2} \tilde{X}_1 \right)^{-1} \tilde{X}_1' M_{\tilde{X}_2} \tilde{y}$$

Limiting Distribution

- The final component required to implement he Hausman test, is an consistent estimate of $V = \text{AsyVar}\left(\sqrt{N}(\widehat{\beta}_1^c - \widehat{\beta}_1^e)\right)$
- As $W_2 \subset W_1$, then $P_2 P_1 = P_2$, and the difference between the consistent and efficient estimators in (7) and (10) is:

$$\widehat{\beta}_1^c - \widehat{\beta}_1^e = A_c^{-1} \tilde{X}_1' P_2 M_{\tilde{X}_2} M^* \tilde{y} \tag{11}$$

- The residual maker M^* is defined as:

$$M^* = I - M_{\tilde{X}_2} P_1 \tilde{X}_1 A_e^{-1} \tilde{X}_1' P_1 M_{\tilde{X}_2}$$

Limiting Distribution

- Given that all variables are group de-meaned, and $QQ = Q$, replacing $\tilde{y}$ with v in (11) leads to the same result:

$$\sqrt{N}\left(\widehat{\beta}_1^c - \widehat{\beta}_1^e\right) = \left(\frac{1}{N}A_c\right)^{-1} \frac{1}{\sqrt{N}} \tilde{X}_1' P_2 M_{\tilde{X}_2} M^* v$$

- Letting $\Omega = E\left[vv' \mid W\right]$, then under H_0, the covariance matrix of the limiting distribution is:

$$V = \left(\operatorname{plim} \frac{1}{N}A_c\right)^{-1} S \left(\operatorname{plim} \frac{1}{N}A_c\right)^{-1} \qquad (12)$$

where the middle term:

$$S = \lim_{N \to \infty} \frac{1}{N} E\left[\tilde{X}_1' P_2 M_{\tilde{X}_2} M^* \Omega M^* M_{\tilde{X}_2} P_2 \tilde{X}_1\right] \qquad (13)$$

Variance Estimation: IID Errors

- Define the estimator of S:

$$\widehat{S} = \frac{1}{N}\tilde{X}_1' P_2 M_{\tilde{X}_2} M^* \widehat{\Omega} M^* M_{\tilde{X}_2} P_2 \tilde{X}_1 \qquad (14)$$

- Different choices of $\widehat{\Omega}$ lead to different estimators of V in (12). Assuming errors are independent across units:

$$\widehat{\Omega} = \mathrm{diag}(\widehat{\Omega}_1, \ldots, \widehat{\Omega}_N).$$

- Under the variance components model $u_i \sim \mathrm{iid}(0, \sigma_u^2)$, $\epsilon_{it} \sim \mathrm{iid}(0, \sigma_\epsilon^2)$:

$$\widehat{\Omega}_i = \widehat{\sigma}_\epsilon^2 I_{T_i} + \widehat{\sigma}_u^2 e_i e_i'.$$

- Estimates of σ_ϵ^2 and σ_u^2 are obtained from residuals of the group-demeaned model, following `xthtaylor`.

Variance Estimation: Cluster-Robust

- An alternative is to use an estimator that allows for clustering by unit or group and is robust to arbitrary serial correlation of errors within clusters and heteroskedasticty across clusters.

Clustering at the unit level

- Let $\widehat{v}_i$ be the $T_i \times 1$ vector of residuals for unit i. Then:

$$\widehat{\Omega}_i = \widehat{v}_i \widehat{v}_i'.$$

Clustering at the group level

- Let $\widehat{v}_g$ be the stacked residuals for all units in group g. Then:

$$\widehat{\Omega}_g = \widehat{v}_g \widehat{v}_g'.$$

Auxiliary Regressions

- Following Davidson and MacKinnon (1993), the Hausman test can also be implemented using auxiliary regressions.
- When testing all variables in x_1 for endogeneity, this requires augmenting the original model with residuals from regressions of each x_1 variable on the instruments in W_2 together with the group dummies. The model is then estimated by pooled OLS
- As before, it is again easier to work with the model where the group dummies have been partialled out from the variables

$$\tilde{y} = \tilde{X}_1' \beta_1 + \tilde{X}_2' \beta_2 + M_2 \tilde{X}_1 \delta_1 + \tilde{\eta}. \qquad (15)$$

Auxiliary Regressions

- We are interested in $\widehat{\delta}_1$. Applying the FWL theorem:

$$\widehat{\delta}_1 = -\left(\tilde{X}_1' P_2 M_{\tilde{X}} P_2 \tilde{X}_1\right)^{-1} \tilde{X}_1' P_2 M_{\tilde{X}_2} M^* \tilde{y}$$

- The second term is exactly what appears in equation (11) for the vector of contrasts $\widehat{\beta}_1^c - \widehat{\beta}_1^e$, hence by substitution:

$$\widehat{\delta}_1 = -A_\delta^{-1} A_c \left(\widehat{\beta}_1^c - \widehat{\beta}_1^e\right) \tag{16}$$

- As the A-terms in (16) cancel in the Wald statistic, testing $\delta_1 = 0$ is equivalent to testing $\text{plim}(\widehat{\beta}_1^c - \widehat{\beta}_1^e) = 0$. When the exogenous controls x_2 are all time-invariant, the consistent and efficient estimators reduce to unit and group FE.

Auxiliary Regressions

- The auxiliary regression in (15) provides the control function representation of the IV estimator, and in this set-up the OLS estimator is unit-FE: $\widehat{\beta}_1 = \widehat{\beta}_1^{FE(i)}$.

- It can also be shown that $\widehat{\delta}_1$ equals the difference between a between-effects regression (including G group dummies) and the unit fixed effects estimator:

$$\widehat{\delta}_1 = \widehat{\beta}_1^{BE} - \widehat{\beta}_1^{FE(i)}. \tag{17}$$

- Thus, while the joint test is the same as testing the equality of the full vector of unit and group FE contrasts, the t-tests on individual δ_{1k} components, compare BE with unit-FE.

- The comparison provided by `xtfelevel` is for unit-group fixed effects, and may therefore lead to different conclusions.

Mundlak Regression

- It is easy to show that the control function regression in (15) is also the Mundlak device. The residual maker is:

$$M_2\tilde{X}_1 = \tilde{X}_1 - P_2\tilde{X}_1$$

- And from (8), we know that

$$P_2\tilde{X}_1 = Q^u\tilde{X}_1 + P_{M^u\tilde{X}_2}M^u\tilde{X}_1$$

hence by substitution:

$$M_2\tilde{X}_1 = \left(I - P_{M^u\tilde{X}_2}\right)M^u\tilde{X}_1 \tag{18}$$

- These are the residuals from separate between-effects regressions of $\bar{x}_{1ki}$ on $\bar{x}_{2i}$ including G group dummies, with each mean replicated T_i times in the data.

Mundlak Regression

- As the residuals are time-invariant for a given individual, for observation *it*, the residual for the *k*-th variable is

$$\left(M_2\tilde{X}_1\right)_{itk} = \bar{x}_{1ki} - \bar{x}_{2i}'\widehat{\pi}_k^{BE} - d_i'\widehat{\theta}_k$$

- Plugging these into the auxiliary model for y_{it} and letting $d_i'\widehat{\theta}_k$ be subsumed into the overall group-effect yields:

$$y_{it} = x_{1it}'\beta_1 + x_{2it}'\beta_2 + \sum_{k=1}^{K_1}\left(\bar{x}_{1ki} - \bar{x}_{2i}'\widehat{\pi}_k^{BE}\right)\delta_k + d_i'\lambda + \eta_{it} \quad (19)$$

- The model can be estimated by OLS including *G* group dummies, or by group fixed effects, which is OLS applied to the group within transformed model.

Mundlak Equivalent of Efficient IV Estimator

- When all variables in x_1 are treated as endogenous, and the variables in x_2 are all time-invariant, there is no need to partial out their means (unless they are of direct interest), since their correlation with x_1 is absorbed in β_2.
- The situation is different once some regressors are treated as exogenous. Consider the efficient IV estimator where x_{1A} is an exogenous policy variable, and x_{1B} is a single endogenous regressor (assume x_2 is empty).
- As illustrated in (19), the IV estimator is equivalent to a Mundlak regression in which the mean of $\bar{x}_{1A}$ is partialled out from the mean of the endogenous regressor $\bar{x}_{1B}$:

$$y_{it} = x_{1it}^{'}\beta_1 + x_{2it}^{'}\beta_2 + \left(\bar{x}_{1Bi} - \bar{x}_{1Ai}^{'}\widehat{\pi}^{BE}\right)\delta_{1B} + d_i^{'}\lambda + \eta_{it}$$

Hybrid Estimator and Potential Bias

- The IV estimator therefore imposes a restriction on the implied coefficient of $\bar{x}_{1A}$ in the usual Mundlak model:

$$\delta_{1A} = -\delta_{1B}\pi^{BE}. \tag{20}$$

- Papke and Wooldridge (2023) suggest that if the unrestricted estimate $\widehat{\delta}_{1A}$ in the usual Mundlak regression is not significant, it can be dropped, and this hybrid estimator may yield more precise estimates of β_{1A} by reducing collinearity with x_{1Ait}.
- But if x_{1A} has little within-unit variation, insignificance of $\bar{x}_{1A}$ may simply reflect low power rather than true exogeneity, and dropping it risks omitted variable bias.

Hybrid Estimator and Potential Bias

- It can be shown that the hybrid estimator converges to a weighted average of the FE and between-effects estimators, where the weights depend on the share of within and between variation in the regressors.

- Since the between-effects estimator is inconsistent when x_{1B} is endogenous (because u_i is in the error term), the resulting mixture (i.e., the hybrid) will also be inconsistent.

- Thus, unless the excluded means are uncorrelated with the means that remain in the model (i.e. all $\pi_k^{BE} = 0$), omitting them rather than partialling them out risks potential omitted variables bias.

xtfelevel

Robust Hausman tests for choosing the level of fixed effects in linear panel data models

xtfelevel *depvar varlist* [*if*] [*in*] *group(varlist)* [tvendog(*varlist*) tvexog(*varlist*) sigmamless sigmamore vce(*vcetype*)]

---

group(*varlist*) variables that define groups that nest units.

tvendog(*varlist*) time-varying regressors always treated as correlated with the unit-specific effect.

tvexog(*varlist*) time-varying regressors always treated as uncorrelated with the unit-specific effect.

sigmamless use residuals from the consistent estimator to estimate the covariance matrix for the Hausman test.

sigmamore use residuals from the efficient estimator to estimate the covariance matrix for the Hausman test.

xtfelevel: Defining Groups

- Groups do not have to be defined by a single identifier. They can be formed additively from several components, as long as units are nested within groups.
- Example (automotive data):
  - **Unit:** model–version–engine size–transmission–AC.
  - **Group (broad):** model–version only.
  - **Group (additive):** model–version + engine size + transmission + AC.
- xtfelevel implements both cases. The additive case uses the method of alternating projections

Simulated Data

- Data is simulated from the following model, setting $G = 100$ groups, each containing 10 units observed over $T = 5$ periods.

$$
\begin{aligned}
y_{it} &= 1 + x_{1Ait} + x_{1Bit} + x_{2i} + u_i + h_{g(i)} + \epsilon_{it} \\
u_i &\sim N(0,1) \\
\epsilon_{it} &\sim N(0,1)
\end{aligned}
$$

- x_{1Bit} is endogenous and correlated with u_i; whereas x_{1Ait} is exogenous but correlated with x_{1Bit}.
- The policy variable of interest is x_{1Ait}, which shows little within-unit variation (time-varying in only 20% of units).
- Group fixed effects estimator is inconsistent for $\beta_{x_{1A}}$, because x_{1Ait} is correlated with x_{1Bit}, and x_{1Bit} is endogenous.

Testing X_{1A} and X_{1B} (H_0 : Unit FE = Group FE)

```
. xtfelevel y x1A x1B x2,  group(idg) vce(cluster idg)
                                                Number of obs     =      5,000
Unit variable:   idu                            Number of units   =      1,000
Group variable: idg                             Number of groups  =        100

------------------------------------------------------------------------------
             |               Robust
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
Efficient    |
         x1A |   1.459548     .02643    55.22   0.000     1.407746     1.51135
         x1B |   1.470631   .0138896   105.88   0.000     1.443408    1.497855
          x2 |   .9712746   .0221096    43.93   0.000     .9279406    1.014609
-------------+----------------------------------------------------------------
Consistent   |
         x1A |   .6029016   .3161743     1.91   0.057    -.0167886    1.222592
         x1B |   1.015079   .0160868    63.10   0.000     .9835499    1.046609
          x2 |   .9629337   .0361778    26.62   0.000     .8920266    1.033841
-------------+----------------------------------------------------------------
Diff_Tested  |
         x1A |  -.8566462   .3622874    -2.36   0.018    -1.566717   -.1465759
         x1B |  -.4555519   .0491084    -9.28   0.000    -.5518026   -.3593013
------------------------------------------------------------------------------
Endogeneity Test:                                                      86.2027
Variables Tested:   x1A x1B                     Chi-sq(2) P-val   =     0.0000
------------------------------------------------------------------------------
```

Testing X_{1A} (H_0 : Unit FE = Group FE, X_{1B} Endogenous)

```
. xtfelevel y x1A x1B x2, tvendog(x1B) group(idg) vce(cluster idg)
                                                Number of obs     =        5,000
Unit variable:   idu                            Number of units   =        1,000
Group variable: idg                             Number of groups  =          100
-------------------------------------------------------------------------------
             |               Robust
           y |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+-----------------------------------------------------------------
Efficient    |
         x1A |   1.007809   .0355045    28.39   0.000     .9382216    1.077397
         x1B |   1.014763   .0161035    63.02   0.000     .9832011    1.046326
          x2 |   .9662914   .0315063    30.67   0.000     .9045402    1.028043
-------------+-----------------------------------------------------------------
Consistent   |
         x1A |   .6029016   .3161743     1.91   0.057    -.0167886    1.222592
         x1B |   1.015079   .0160868    63.10   0.000     .9835499    1.046609
          x2 |   .9629337   .0361778    26.62   0.000     .8920266    1.033841
-------------+-----------------------------------------------------------------
Diff_Tested  |
         x1A |  -.4049075   .3176995    -1.27   0.202    -1.027587    .2177721
-------------+-----------------------------------------------------------------
Diff_Endog   |
         x1B |   .0003161    .000248     1.27   0.202      -.00017    .0008023
-------------------------------------------------------------------------------
Endogeneity Test:                                                        1.6243
Variables Tested:   x1A                         Chi-sq(1) P-val   =      0.2025
-------------------------------------------------------------------------------
```

Mundlak Representation: Testing X_{1A} and X_{1B}

```
.  *estimate mundlak regression - estimates are unit FE´s
. reghdfe y x1A x1B x2 mui_x1A mui_x1B, a(idg) vce(cluster idg)

HDFE Linear regression                            Number of obs   =      5,000
Absorbing 1 HDFE group                            F(   5,      99) =    3463.00
Statistics robust to heteroskedasticity           Prob > F        =     0.0000
                                                  R-squared       =     0.9096
                                                  Adj R-squared   =     0.9077
                                                  Within R-sq.    =     0.8264
Number of clusters (idg)     =        100         Root MSE        =     1.0662

                                 (Std. Err. adjusted for 100 clusters in idg)
------------------------------------------------------------------------------
             |               Robust
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         x1A |   .6029016   .3162692     1.91   0.060    -.0246452    1.230448
         x1B |   1.015079   .0160916    63.08   0.000     .9831502    1.047009
          x2 |   .9749439    .020648    47.22   0.000     .9339739    1.015914
     mui_x1A |   1.189611   .3181731     3.74   0.000     .5582867    1.820936
     mui_x1B |   .7901667   .0242087    32.64   0.000     .7421315     .838202
       _cons |   1.040708   .0058963   176.50   0.000     1.029009    1.052408
------------------------------------------------------------------------------

. testparm mui_x1A mui_x1B

 ( 1)  mui_x1A = 0
 ( 2)  mui_x1B = 0

       F(  2,    99) =  538.21
            Prob > F =    0.0000
```

Mundlak Equivalent of Efficient IV Estimator

```
.  *partial out mean of x1A and x2 from x1B
. reghdfe mui_x1B mui_x1A x2, a(idg) resid
. predict mui_x1B_partial, res


.  *estimate adjusted mundlak
. reghdfe y x1A x1B x2 mui_x1B_partial, a(idg) vce(cluster idg)

HDFE Linear regression                            Number of obs   =      5,000
Absorbing 1 HDFE group                            F(   4,      99) =    4328.09
Statistics robust to heteroskedasticity           Prob > F        =     0.0000
                                                  R-squared       =     0.9096
                                                  Adj R-squared   =     0.9077
                                                  Within R-sq.    =     0.8264
Number of clusters (idg)     =        100         Root MSE        =     1.0662

                                   (Std. Err. adjusted for 100 clusters in idg)
--------------------------------------------------------------------------------
                |               Robust
              y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
----------------+---------------------------------------------------------------
            x1A |   1.007809   .0252821    39.86   0.000      .957644    1.057974
            x1B |   1.014763   .0161051    63.01   0.000     .9828074    1.046719
             x2 |   .9662914   .0206452    46.80   0.000     .9253269    1.007256
mui_x1B_partial |   .7904828   .0242593    32.58   0.000      .742347    .8386187
          _cons |   1.185564   .0056486   209.89   0.000     1.174356    1.196772
--------------------------------------------------------------------------------
```

Hybrid Estimator: No Partialling Out (Illustration Only)

```
. reghdfe y x1A x1B x2 mui_x1B, a(idg) vce(cluster idg)

HDFE Linear regression                            Number of obs   =      5,000
Absorbing 1 HDFE group                            F(   4,      99) =    4346.80
Statistics robust to heteroskedasticity           Prob > F        =     0.0000
                                                  R-squared       =     0.9094
                                                  Adj R-squared   =     0.9075
                                                  Within R-sq.    =     0.8260
Number of clusters (idg)     =        100         Root MSE        =     1.0672

                                   (Std. Err. adjusted for 100 clusters in idg)
------------------------------------------------------------------------------
             |               Robust
           y |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         x1A |   1.788236   .0267531    66.84   0.000     1.735152     1.84132
         x1B |   1.016747   .0159982    63.55   0.000     .9850031    1.048491
          x2 |   .9749033   .0206466    47.22   0.000      .933936     1.01587
     mui_x1B |   .7865974   .0240587    32.69   0.000     .7388598     .834335
       _cons |   1.041618   .0058695   177.46   0.000     1.029971    1.053264
------------------------------------------------------------------------------
```

Conclusion

- This presentation has set out a Hausman-type test for deciding whether group-level fixed effects are sufficient, compared to unit fixed effects.
- The approach builds on Papke and Wooldridge (2023) to a general IV framework, allowing some regressors to be treated as endogenous under both the null and the alternative.
- The test can be implemented using the Stata command `xtfelevel`, which provides robust and cluster-robust covariance estimators.
- The efficient IV estimator is equivalent to a Mundlak form, where the unit-means of exogenous variables are partialled out of the means of the endogenous regressors.

References

Basu, D. 2023. The Yule-Frisch-Waugh-Lovell theorem for linear instrumental variables estimation. *arXiv preprint arXiv:2307.12731.* .

Davidson, R., and J. G. MacKinnon. 1993. *Estimation and inference in econometrics*, vol. 63. New York: Oxford.

Mundlak, Y. 1978. On the pooling of time series and cross section data. *Econometrica* 46: 69–85.

Papke, L. E., and J. M. Wooldridge. 2023. A simple, robust test for choosing the level of fixed effects in linear panel data models. *Empirical Economics* 64(6): 2683–2701.

Wooldridge, J. 2019. Correlated random effects models with unbalanced panels. *Journal of Econometrics* 211(1): 137–150.